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THE ECONOMICAL PARAMETER SURVEY PROGRAM FOR LONG-TERM TRANSIENT TEMPERATURE DISTRIBUTION AND STRESS ANALYSIS AROUND THE SODIUM LEVEL OF THE LIQUID METAL FAST BREEDER REACTOR*

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The thermal stress around the sodium level of the LMFBR reactor vessel, caused by the axial temperature distribution during heat-up and cool-down transient conditions, is one of the most important problems for a reactor vessel designer, especially in a conceptual design stage. Key parameters relating to plant operating transient conditions and reactor vessel main configuration should be settled carefully, thermal stresses not being allowed to exceed design stress limits. In order to examine the sodium level thermal stress easily and economically, a simple computer program for parameter survey has been developed. This program uses a one-dimensional Fourier series solution in transient temperature distribution analysis, and an analytical stress solution based on shell theory in stress value estimation. This paper presents a simplified and economical calculation method for the axial temperature distribution and stress value in the LMFBR operating transient conditions, and analysis examples obtained with this computer program.

The LMFBR reactor vessel is made of stainless steel, a material with low thermal conductivity. The upper part of a typical reactor vessel is shown in Fig. 1. The sodium surface is covered with Ar gas and the outer surface of the reactor vessel is coated with thermal insulation.

Under the plant operating transient conditions, for example plant heat-up, the sodium temperature rises by about 300° in 30 hours or so. Because of the low thermal conductivity of stainless steel, the reactor vessel wall temperature above the sodium surface level does not change so rapidly and this leads to a severe axial temperature distribution and to high thermal stress around the sodium surface level of the vessel.

To analyse the operating transient temperature distribution in the vessel wall, a simple model was made. As the vessel diameter is two or three hundred times as

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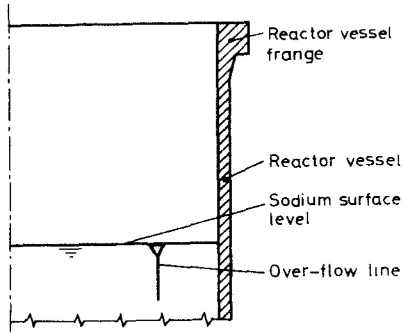

Fig. 1 Conception of reactor vessel upper part

large as the wall thickness, a one-dimensional model is acceptable in the axial temperature distribution analysis. The effect of temperature distribution in the wall thickness direction on the thermal stress is neglected, for in a long transient condition the temperature change between the inner side and the outer side of the vessel wall is small.

Other effects, such as gamma-ray heating, heat loss from the vessel wall and thermal radiation, are also neglected. These neglected factors will be taken into account in the detailed design stage, using FEM, etc.

The temperature transient analysis model is simplified as mentioned above, and thus it becomes possible to use an analytical solution; this enables designers to analyse the temperature distribution without regard to the time increment.

The thermal stress calculation is based on the analytical solution given by Timoshenko's shell theory [1].

The use of analytical solutions is convenient for a designer who wants to analyse many cases in a short time with the fewest input data in the initial design study.

The advantage of this computer program is that it simplifies the data required to analyse a problem. Design is by nature an interactive process. Designers try to find an optimum solution to the problem, and in this process the computer program can relieve designers of the laborious task of data preparation and shorten the time to obtain a reasonable solution for given key parameters of the operating conditions.

Temperature distribution analysis method

As described above, the analysis model is one-dimensional. The temperature at the top of the reactor vessel flange is assumed to be constant. The reactor vessel wall region in contact with the sodium is assumed to have the same temperature as the sodium coolant. Thus, the subject of analysis is the reactor vessel wall above the sodium level. Material characteristics are treated as temperature-independent.

The initial temperature distribution is taken into account in the analysis model by multi-linear approximation. Temperature change with time is also modeled by a multilinear approximation.

The one-dimensional transient heat conduction equation is written as follows:

$$\frac{\partial u}{\partial t} = \varkappa \frac{\partial^2 u}{\partial x^2} \tag{1}$$

Where u is temperature, t is time and x is the distance from the sodium level. $\varkappa$ is thermal diffusivity.

Boundary conditions and initial conditions are expressed in the following equations; the notation l is the length between the top of reactor vessel flange and the sodium level:

$$\begin{aligned} u(O, t) &= f_0(t) \\ u(l, t) &= T_f \\ u(x, O) &= g_0(x) \end{aligned} \tag{2}$$

$f_0(t)$ and $g_0(t)$ satisfy the following relationships:

$$\begin{aligned} g_0(l) &= T_f \\ f_0(O) &= g_0(O) = T_t \end{aligned} \tag{3}$$

$f(t)$ and $g(t)$ are defined in the following equations:

$$\begin{aligned} f(t) &= f_0(t) - T_l \\ g(x) &= g_0(x) - T_l \end{aligned} \tag{4}$$

The solution of Eq. (1) under the conditions (2) is expressed as follows:

$$u(x, t) = T_l + \frac{2}{l} \sum_{n=1}^{\infty} \exp\left(-\varkappa \frac{n^2\pi^2}{l^2} t\right) \cdot \sin\left(\frac{n\pi x}{l}\right) \cdot \left[\int_0^l g(\xi) \sin\left(\frac{n\pi\xi}{l}\right) d\xi + \frac{n\pi\varkappa}{l} \int_0^t \exp\left(\varkappa \frac{n^2\pi^2}{l^2} \tau\right) \cdot \{f(\tau) + (-1)^n (T_l - T_f)\}\, d\tau \right] \tag{5}$$

The initial temperature distribution modelled by the multi-linear approximation is shown in Fig. 2.

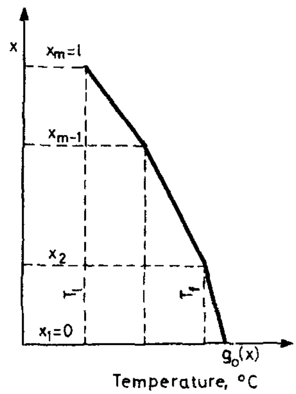

Fig. 2 Initial temperature distribution model

The temperature history of the sodium coolant is also expressed by a multi-linear approximation, as shown in Fig. 3. There are two integration parts in Eq. (5); one is related to the initial temperature distribution and the other to the sodium temperature change. Through the use of the multi-linear approximation, these two integration parts can be analytically integrated.

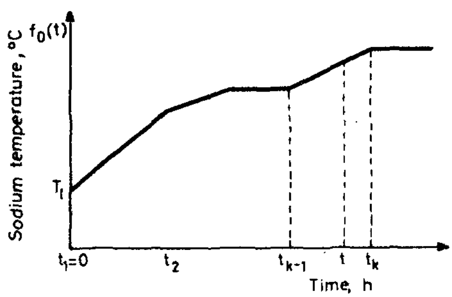

Fig. 3 Sodium temperature change model

The final solution is Eq. (6):

$$u(x,t) = T_l + \frac{2}{l}\sum_{n=1}^{\infty} \exp\left(-\varkappa\frac{n^2\pi^2}{l^2}t\right)\sin\left(\frac{n\pi}{l}x\right)\cdot$$

$$\left[\sum_{i=2}^{m}\left\{(A_i + B_i x_i)_i\left(\frac{-l}{n\pi}\right)\cos\left(\frac{n\pi}{l}x_i\right)\right.\right.$$

$$\left.-(A_i + B_i x_{i-1})\left(\frac{-l}{n\pi}\right)\cos\left(\frac{n\pi}{l}x_{i-1}\right)\right. \tag{6}$$

$$+B_i\frac{l^2}{n^2\pi^2}\left(\sin\left(\frac{n\pi}{l}x_i\right)-\sin\left(\frac{n\pi}{l}x_{i-1}\right)\right)\Bigg\}$$

$$+\sum_{j=2}^{k}\frac{l}{n\pi}\left\{\exp\left(\varkappa\frac{n^2\pi^2}{l^2}t_j\right)\left(C_j+D_j\left(t_j-\frac{l^2}{\varkappa n^2\pi^2}\right)\right)\right.$$

$$\left.-\exp\left(\varkappa\frac{n^2\pi^2}{l^2}t_{j-1}\right)\left(C_j+D_j\left(t_{j-1}-\frac{l^2}{\varkappa n^2\pi^2}\right)\right)\right\}\Bigg]$$

where the notations, A_i, B_i, C_i and D_j are expressed as follows:

$$A_i = g(x_{i-1})-B_i\frac{x_{i-1}}{x_i-x_{i-1}}$$

$$B_i = \frac{g(x_i)-g(x_{i-1})}{x_i-x_{i-1}}$$

$$C_j = f(t_{j-1})-D_jt_{j-1}+(-1)^n(T_i-T_f) \qquad (7)$$

$$D_j = \frac{f(t_j)-f(t_{j-1})}{t_j-t_{j-1}}$$

If time t is bigger than t_{k-1} and less than or equal to t_k, the last t_j value in Eq. (6) must be time t.

The temperature distribution calculation is based on Eqs (6) and (7). Equation (6) is expressed using Fourier series and the solution of the equation does not converge quickly. In order to get a well-converged solution, it is necessary to calculate many Fourier terms n. From observation of the numerical result, it was noticed that the temperature value at x and t varied against n slowly in a decay oscillation manner.

From the viewpoint of computer cost, it is not reasonable to calculate many terms. Accordingly, an average method that enables us to get a reasonable solution without greatly increasing the computing time is adopted.

The average method is explained as follows. Let u_m be the solution which has m Fourier terms. The final solution is expressed as the average value of some u_m solutions as in the following equation:

$$u(x,t) = \left(\sum_{k=1}^{m} u_{n+k}\right)\Big/ m \qquad (8)$$

The calculation example for uniform heat-up conditions is shown in Fig. 4.

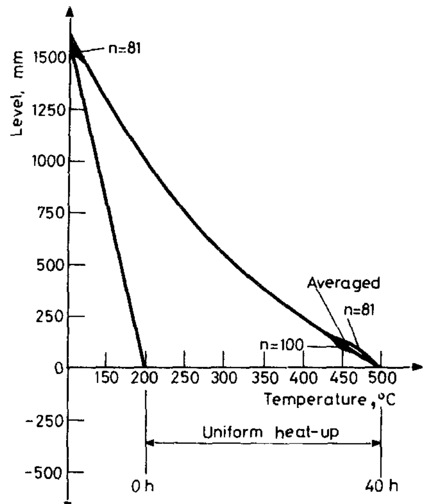

Fig. 4 Temperature distribution analysis results

Stress distribution analysis method

If there is an axial temperature gradient change in a cylinder, as shown in Fig. 5, thermal stress is induced in the cylinder wall. Such thermal stress is given analytically by Timoshenko's shell theory. When the axial temperature gradient

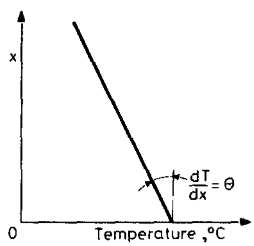

Fig. 5 Temperature gradient change model

changes at $x=0$, longitudinal stress ϱ_x and hoop stress ϱ_θ are expressed by the following equations:

$$\varrho_x(x) = \mp \frac{\{3(1-\nu^2)\}^{1/4}}{4(1-\nu^2)} E\alpha\sqrt{ah}\,\varphi(|\beta x|)\frac{\mathrm{d}T}{\mathrm{d}x} \tag{9-1}$$

$$\varrho_\theta(x) = \frac{1}{12}\left\{\frac{27}{(1-\nu^2)}\right\}^{1/4} E\alpha\sqrt{ah}\,\psi(|\beta x|)\frac{\mathrm{d}T}{\mathrm{d}x} \mp \nu\sigma_x(x) \tag{9-2}$$

where the notations are as follows:

E = Young's modulus ν = Poisson's ratio
α = thermal expansion coefficient
a = mean radius of a cylinder
h = cylinder wall thickness
$\varphi(y) = e^{-y}(\cos y + \sin y)$
$\psi(y) = e^{-y}(\cos y + \sin y)$
$\beta = \left\{\frac{3(1-\nu^2)}{a^2h^2}\right\}^{1/4}$

dT/dx means the temperature gradient change at $x = 0$, as shown in Fig. 5. The sign $\mp$ means negative for outer-surface stress and positive for inner-surface stress in a cylinder.

Equations (9–1) and (9–2) were modified to be applied to more complicated axial temperature distribution. If the number of temperature gradient discontinuity points is n, the stresses are expressed by the following equations:

$$\begin{aligned} \sigma_x(x) &= \mp A \cdot \sum_{i=1}^{n} \varphi(|\beta(x-l_i)|) \cdot \theta_i \\ \sigma_\theta(x) &= B \cdot \sum_{i=1}^{n} \psi(|\beta(x-l_i)|) \cdot \theta_i \mp \nu\sigma_x(x) \end{aligned} \tag{10}$$

Where l_i is the position of a point where the temperature gradient change value is θ_i. A and B are defined by the following equations:

$$A = \frac{\{3(1-\nu^2)\}^{1/4}}{4(1-\nu^2)} E\alpha\sqrt{ah}$$

$$B = \frac{1}{12}\left\{\frac{27}{(1-\nu^2)}\right\}^{1/4} E\alpha\sqrt{ah}$$

Examples of analysis of operating transient conditions

Temperature distribution analysis results from this parameter survey program were compared with the results obtained with the FEM program NASTRAN. They agreed well with each other, as shown in Fig. 6. Figure 7 shows a comparison of the results of stress distribution on the cylinder outer surface.

The data used in the calculation are as follows:

Thermal diffusivity $\varkappa$ = 4.80 mm^2/s
Young's modulus E = 17,200 kg/mm

Poisson's ratio	ν	$= 0.3$
Thermal expansion coefficient	α	$= 19.57 \times 10^{-6}$ 1/°C
Outer radius of cylinder	a	$= 4000$ mm
Wall thickness of cylinder	h	$= 30$ mm

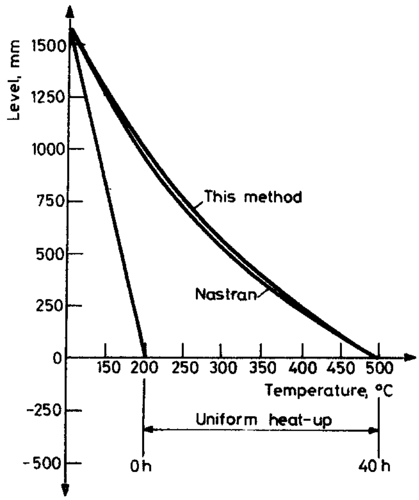

Fig. 6 Comparison of temperature calculation results

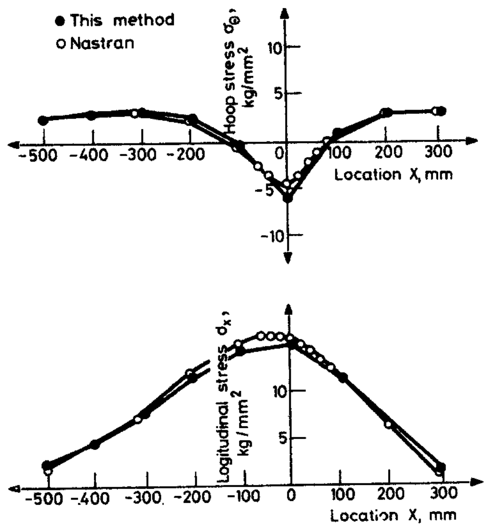

Fig. 7 Comparison of stress component values

On the basis of the same data, a parameter survey on heat-up transient conditions was conducted. The maximum stress intensity values for each heat-up condition are illustrated in Fig. 8. This Figure shows that there is a heat-up condition which generates minimum stress in a cylinder when the heat-up time is restricted to some constant value.

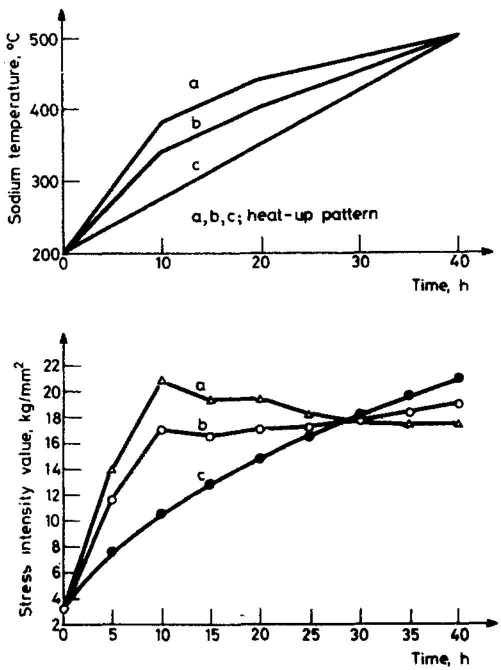

Fig. 8 Heat-up pattern and corresponding stress intensity value

Conclusion

In the early design stage of the LMFBR reactor vessel, planning on minimizing stress around the sodium surface region during long operating thermal transient conditions is an important concern for designers.

In order to survey and fix plant operating conditions which keep the thermal stress caused by the axial temperature distribution in a reactor vessel within the stress limits, a simple and economical computer program has been developed. This computer program uses a one-dimensional transient heat conduction analytical solution and a theoretical stress solution given by the thin shell theory.

The calculation results obtained with this code are reasonable enough to reveal the relation between long operating transient conditions and thermal stresses. As this program is based on an analytical solution, the required input data volume is smaller than in FEM, and this is the merit of the parameter survey program in the conceptual design stage of the LMFBR reactor vessel.

Reference

1 S. P. Timoshenko and S. Woinowski-Krieger, Theory of Plates and Shells, 2nd Ed., McGraw-Hill Book Company Inc., New York.

Zusammenfassung — Der durch die axiale Temperaturverteilung während der Übergangsphase von Aufheizen und Abkühlen verursachte thermische Streß rund um den Natriumfüllstand des LMFBR-Reaktorkessels ist für den Reaktorkesselkonstrukteur, besonders im Entwurfstadium eines der wichtigsten Probleme. Die mit den transienten Operationsbedingungen der Anlage und der Gestaltung des Reaktorkessels in Beziehung stehenden Grundparameter müssen sorgfältig ermittelt werden, um zu verhindern, daß der thermische Streß die geplanten Streßgrenzen nicht überschreitet. Um den thermischen Streß um den Natriumfüllstand, leicht und ökonomisch zu bestimmen, wurde ein Komputerprogramm für die Parameterübersicht aufgestellt. In diesem Programm wird von einer eindimensionalen Fourrierreihe zur Analyse der transienten Temperaturverteilung und einer auf der Schalentheorie von Streßwertbestimmungen basierende analytische Lösung Gebrauch gemacht. Im vorliegenden Artikel werden eine vereinfachte und wirtschaftliche Berechnungsmethode für die axiale Temperaturverteilung und Streßwerte bei transienten Arbeitsbedingungen mitgeteilt und mit diesem Komputerprogramm ausgeführte Analysenbeispiele angegeben.

Резюме — Термическое напряжение, возникающее вокруг уровня натрия в баке жидкометаллического быстрого реактора-размножителя (ЛМФБР) и вызванное аксиальным распределением температуры в условиях нагрева и охлаждения, является наиболее важной проблемой для конструкторов ядерных реакторов, в особенности, на стадии проектирования. Ключевые параметры, касающиеся эксплуатационных нестационарных условий и основной конфигурации бака ядерного реактора, должны быть тщательно установлены, а исследуемые термические напряжения не должны превышать установленные пределы. С целью простого и экономичного определения термического напряжения уровня натрия, разработана простая программа, которая использует решение одноразмерных фурье-рядов при анализе нестационарного температурного распределения, а аналитическое решение паряжение основано на теории оболочек при определении величины напряжения. Представлен упрощенный и экономичный метод расчета аксиального распределения температуры и величины напряжения в реакторе ЛМФБР, а также показаны примеры анализа, проведенного с помощью компьютерной программы.